Mathematical Analysis to Fight COVID-19 Pandemic

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Abstract: The outbreak of COVID-19 (Corona Virus Disease of 2019) has claimed numerous lives worldwide and there is an urgent need to find solutions to deal with the pandemic situation. Towards this, mathematical modeling can be utilized to help calculate and predict the pandemic curve by considering different parameters that contribute in the transmissibility of the contagious infection, such that it can be controlled. One such metric that can prove to be useful is R_0-basic reproduction number, the calculation for which has been modeled for COVID-19 in this paper. The significance, calculations and the conditions measured by this metric are discussed, along with the various tips for prevention of catching the infectious disease. This directly relates to gauging the contagiousness and thus supporting the various solutions proposed towards dealing with the COVID-19 pandemic.

Keywords and Phrases: Basic Reproduction Number, COVID-19, Infectious Disease Transmission, Mathematical Modeling, Pandemic, R_0

1. INTRODUCTION

The study and analysis of diseases' and health conditions' determining factors and patterns of distributions in defined populations is termed as 'Epidemiology'. This lays the foundation for public health by identifying risks and targets for shaping the policy decisions and practices towards preventive healthcare.To take measures for controlling the spread of infections, along with getting a better understanding of the diseases' epidemiological prototypes, it is effective to mathematically formulatethe disease models. This mathematical modeling can help in estimating the diseases' point of origin and finishing time of its spread, allowing proper decisions regarding control/prevention steps to be taken [1].

A scholar had earlier tried to explain the non-abstract application of singularities of a complex valued function by taking the example of a nuclear bomb attack in which finding the singularities of the bomb's wave or destruction function could help save lives. In mathematics, singularity of a function refers to a point at which the function behaves in contrast to its normal behavior. His argument was ignored at the time considering it as not practical to sit back and calculate for function values in such times of crisis; but taking the case of COVID-19 pandemic situation that the world is in today, the approach is valid and quite relevant, as the infected people are the singularities of the normal life function in this case [2]. This goes on to show that mathematics, 'the queen of all sciences' can be applied to look for solutions in any given problem.

Towards the end of 2019, the COVID-19 virus began to rapidly spread across the world.The severity of the virus and its spread created an alarming situation, leading to a global pandemic. So, for increasing the awareness about the situation, it became necessary to have real-world epidemiological data on the disease [1].

Mathematical modeling and analysis has been leading the front ever since the COVID-19 pandemic broke out. It is the application of mathematics that has found its way in time to time representation of the increasing count of number of infections and thus predicting the pandemic curve. Different parameters showing different levels of involvements are made use of to get better results. Since early 2020, there have been ongoing works estimating the epidemiological parameters, transmission rates and predictions for COVID-19 infections [3].

In this paper, a mathematical metric, called as the basic reproduction number/rate/ratio (R_0) is modeled to describe the transmissibility of the infectious disease of COVID-19. R_0 has been widely adopted for use by epidemiologists as it serves to be very useful in calculations for controlling and predicting the diseases' transmission. Thisdirectly relates to the current problem ofrising COVID-19 infected personsand can be used to propound solutions for dealing with the prevailingpandemic.

2. MATHEMATICAL ANALYSIS OF COVID-19

The whole world is facing crisis owing to the COVID-19 pandemic, which originated in December 2019 in Wuhan and has claimed millions of lives so far. It is essential to recognize its transmission and spread in order to understand its impact. The disease has been considered to be relative of the SARS (severe acute respiratory syndrome). It is unclear what the source of the disease has been and whether a peak has already

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been reached since modern mathematics is based on collaboration and COVID-19 has made that aspect quite difficult. To explore the impact and nature of the transmission, along with prediction of repercussions and future trajectory, mathematical sciences play a vital role. The forecasting of diseases' progression under the effect of intervening health measures can be understood using basic mathematical models by researchers. As the models' complexity grows, questions pertaining to allocation of resources and implementing policies (such as lockdown) can be effectively answered. A mathematical concept- Stochastic process, is one way of studying this spread and with time, analyzing random happenings. Basic calculus can be applied to this process to get a set of differential equations for studying the disease's susceptibility rate in a given population. From there, statistical techniques employing statistics for finding the disease's probability to spread can be developed [4].

These computational equations do not directly equate to health and safety policies, but they offer models which, based on the available data, provide best educated estimates of what might happen in various scenarios. Therefore, through these processes and mathematical models that are built on statistics, calculus and probability, we are not left guessing and have measures which help policy makers get insightabout what strategies around COVID-19 to implement and what effect they will have in the real world. Based on the examined effects of COVID-19, it is too early to sayhow it will influence the new mathematical results' flow, but given the nature of modern mathematics which values social interaction a lot, changes are unavoidable [5].

3. ASSESSING CONTAGIOUS INFECTIONS

A mathematical metric that is widely used for assessing the contagiousness of infections or diseases is R_0 (read as R naught)- the basic reproduction number or ratio or rate. As a disease is transmitted to new people, it is said to have reproduced itself. Using R_0 value, the average number of persons that are susceptible to contracting that disease from someone already infected can be known. This applies specifically to populations which have previously not been vaccinated and are infection-free [6]. For instance, an R_0 value of 10 for a disease would mean that the person having this disease will transmit and infect an average of 10 more people, and this process of replication of disease will continue if there's no person immune to it or vaccinated against it in the community. Therefore, the R_0 value of a disease applies only when no one has had the disease before or been vaccinated against it, and there's no control over its spread, making everyone in the population completely vulnerable to it. Owing to the advances in medicine and healthcare, it is rare to find such combination of conditions for a disease nowadays. As compared to the deadly diseases in the past, most diseases can now be contained, and also cured. For example, the worldwide outbreak of swine flu in 1918 that claimed over 50 million lives was estimated to have R_0 value between 1.4 and 2.8, but when this flu or H1N1 virus broke out in 2009 again, the R_0 value was estimated between1.4 and 1.6 by researchers, and the available vaccines and antiviral drugs made this less fatal [7].

Based on the R_0value, there exist three possibilities of whether the infection/ disease will be potentially transmitted or declined[7]:

- For R_0 value less than 1, every existing infection leads to less than one new infection, which indicates that the disease will decline and eventually die out.
- For R_0 value equal to 1, every existing infection leads toone new infection, which indicates that the disease is stable and will stay alive but not lead to an epidemic.
- For R_0 value more than 1, every existing infection leads to more than one new infection, which indicates that the disease will spread among people and may lead to an epidemic.

4. COVID-19 R_0

According to Emerging Infectious Diseases' online published study, the R_0value for COVID-19 has been calculated as a median of 5.7, which is almost double the earlier estimated value of 2.2 to 2.7; rightly declaring it is as a pandemic. This value of 5.7 indicates that a COVID-19 infected person can transmit the virus to potentially 5-6 persons.Researchers arrived at this value based on the data from Wuhan, China, where the original outbreak happened, and using parameters like the time elapsed between exposure to virus and showing of symptoms- virus incubation period (4.2 days).Also, time duration of 2-3 days was estimated as the doubling time of the virus spread and the time it takes for the related cases' hospitalizations and deaths to double.Shorter this time duration, faster is the pace at which the virus infection is spreading. Given the R_0 value estimate, 82% of the population, at the very least, needs to get immunized to COVID-19 such that its transmission gets controlled [6][7]. Studies suggest that the transmission of the disease can be stopped through vaccinations, herd immunity, quarantining, active monitoring of infected people and measures to ensure physical distancing.

4.1 Calculation of $R_0$of Diseases

In epidemiology, factors that are considered for calculating a disease's R_0 value are [7]:

- Infective period: The time period from the onset of symptoms till they stop showing is referred to as the infectious period. Some diseases are more contagious than others, like the flu in infected persons can stay for more than a week, and for even longer durations in children.The longer a disease's infective period is, the more likely it is that aninfected person can spread it to other people, and the more it will contribute towards a higher value of R_0.

- Contact rate: For a given susceptible population, a disease's effective contact rate is the effective contacts per unit time. If a person who has contracted a contagious disease stays quarantined, the disease's transmission will be slow. The more a person infected with a contagious disease comes in contact with people, who are not vaccinated or infected, the more is the contact rate of that disease, and the more it will contribute to a higher value of R_0.
- Mode of transmission: Diseases can spread through indirect means like suspended air particles, vectors or inanimate objects, or through direct means like droplet spread or direct contact. Those diseases that travel through air are the ones that most easily transmit and spread, such as the flu. On the other hand, diseases that require bodily fluids for transmission aren't easily contracted. So, illnesses that are airborne and don't need direct physical contact to transmit spreadmore and have a higher value of R_0.

Figure 1 illustrates in increasing order of estimated R_0 value, the commonly known diseases.

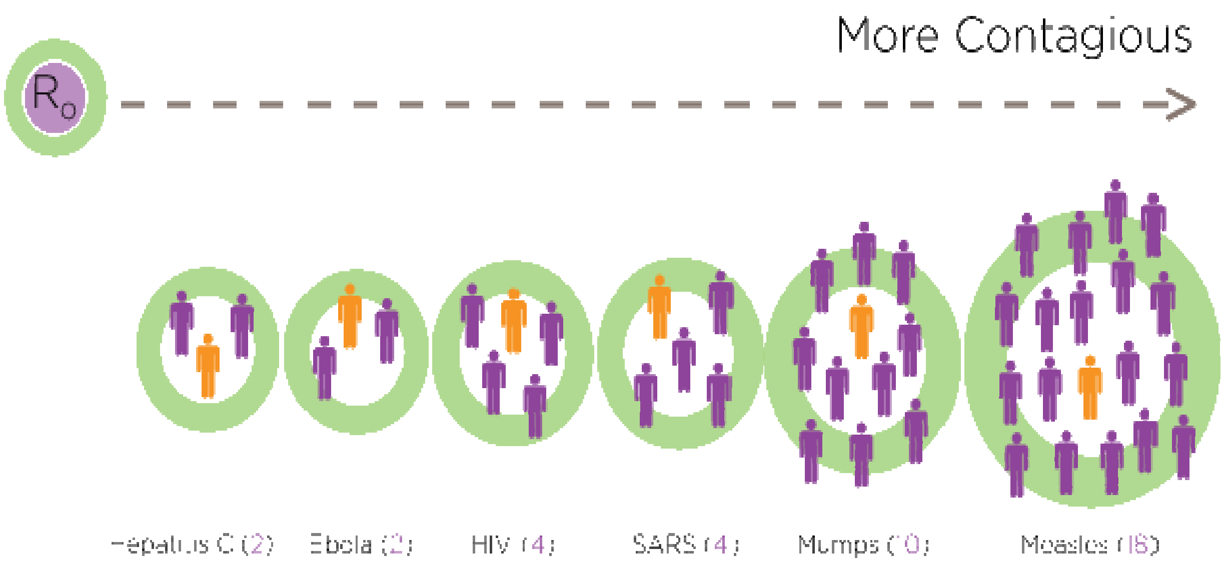

Fig. 1. Increasing infectiousness and R_0 values of diseases [7]

4.2 Conditions Measured by R_0

The R_0 value is not a bacteria or virus caused infectious diseases' intrinsic property and is rather determined by various biological as well as non-biological factors. Simply put, R_0 can be defined to be directly proportional to the average contact rate and the infective period, as shown through equation 1.

$$R_0 = \tau \times c \times d \tag{1}$$

where τ represents the transmissibility, that is the probability of infection spread on contact between an infected and a susceptible person, c is the average effective contact rate, and d isthe infectiousness duration[2].

The model and symbols used are part of differential equation field of mathematics [4], which is beyond the scope of this paper. The numeric value of R_0 can be fixed at 1 and according to the law of Trichotomy, three cases can arise for real analysis:(i) $R_0 = 1$ (ii) $R_0 < 1$ (iii) $R_0 > 1$.

Taking two particular values of $R_0$for thecase (iii), vitality of its process can be interpreted- the value 2 for R_0 of disease can be put numerically as a progression 2, 4, 8, 16, 32, 64… and for the R_0 value 3, the pattern will be 3, 9, 27, 71, 213, 639…which shows that even a slight increase in the R_0 value can blow up the process.

The highest R_0 value of 5.7 was initially attained in Wuhan, China where the COVID-19 virus originated. In India, this R_0 value is calculated weekly by IMSC (Institute of Mathematical Sciences), Chennai. It was alarmingly high and fluctuating in the initial days of the pandemic, followed by a downward trend because of the imposed lockdown, and since its lifting also (unlocking), has maintained a value of less than 1.5 and not much higher [8].The success of the nationwide lockdown in containing the spread of COVID-19 pandemic and its role in the subsequent decrease of R_0 value in the country has been shown in [9].Figure 2 below shows the decline in R_0 value across India.

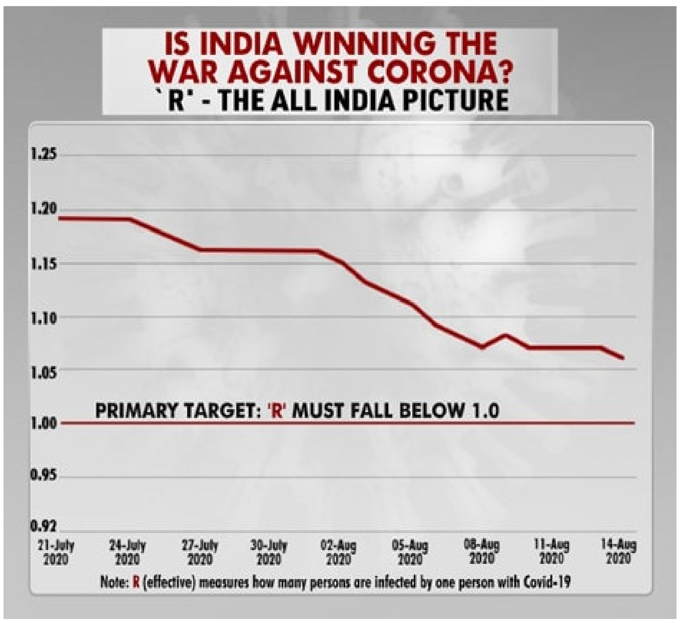

Fig. 2. Average R_0 value in India [8]

So far so good, the curve seems to be flattening. However, the war against COVID-19 is still on and there is no room for complacency; we still need to beware of any new spikes in the curve [8].

4.3 Prevention Tips

Research is ongoing to find cures for various conditions, including the COVID-19, and it is unlikely that infectious diseases will disappear anytime soon. Following are some of the steps that can be followed tohelp in preventing contagious diseases' spread [7]:

- Make yourself aware about the transmission of differentcontagious diseases.
- Ask for vaccinations of different diseases and stay up to date with your routine vaccinations.
- Learn from your doctor about what steps must be ensured on an individual level for stopping further spread of infection.
- Observe physical distancing and maintain hygiene.

4.4 Is R_0measure enough?

At the beginning of any epidemic, it is difficult to get an accurate estimate of R_0because it involves determination of a virus's biological characteristics as well as understanding the rate of contact among people This leads to modellers making assumptions of factors such as the human movement, which limits the precision of the models and hence the generated predictions' accuracy. This uncertainty around R_0 makes some modellers believe it to be a poorly measured metric which is not enough because it can influence and misplace public health actions.Epidemiologists define a slightly different form of the reproductive number R, called the effective reproductive number, represented by R_e. Once the virus is no longer in its initial stages and becomes more common, and measures surrounding public health have commenced, $R_e$captures the transmissions to gauge whether these initiatives are effectively working towards reducing the viral spread or not.This flexible and context-dependent nature of R_e value provides help in getting a more accurate assessment of the contagious disease [10].

5. CONCLUSION

There is continuous advancement in medical science, but contagious diseases are a long way from just vanishing. The corona virus disease of 2019 continues to spread throughout the world, but at comparatively lower rates now. The basic reproduction number value provides a calculation which is helpful in predicting and assisting to control its transmission. It is not the disease's intrinsic property and is determined by various non-biological and biological factors too.Through this paper, the infectiousness and transmissibility of COVD-19 has been gauged, i.e., how the $R_0$is calculated and what all it measures has been explained. Few tips for prevention of contagious diseases, along with possible metric form for accurate estimations of infectiousness are also discussed since there is an on-going battle against COVID-19 worldwide unless the vaccines arrive.

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